INTERNATIONAL JOURNAL FOR NUMERICAL AND ANALYTICAL METHODS IN GEOMECHANICS, VOL. 20, 673–690 (1996)

SHORT COMMUNICATION

THE 'NEW IMPLICIT METHOD' FOR TUNNEL ANALYSIS

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SUMMARY

Tunnel excavation is a coupled three-dimensional problem dealing with two different structures: lining and rockmass. For a simple application it is useful to develop simplified methods by treating the problem as plane strain.

If the problem of tunnel face advance presents an axisymmetric geometry, then we show that the major parameter governing the ground–interface–lining interaction is the convergence of the tunnel U_0 at the moment of the lining installation.

The 'New Implicit Method' (NIM) presented in this paper makes use of principles similar to those of the 'convergence–confinement' method, but it provides a better appreciation of the coupled behaviour of rockmass and lining. For independent time constitutive laws (elasticity and plasticity), we point out that the convergence U_0 depends not only on the mechanical behaviour of the rockmass and on the distance from the tunnel face, as predicted by the 'convergence–confinement' method, but also on the stiffness of the lining previously set.

We present the 'NIM' for elastic and perfect elastoplastic rockmasses without dilatancy for many criteria. The development of this new method is based on the results of tunnel calculations with an axisymmetric FEM numerical model that takes into account the three-dimensional aspect of the problem.

Using this method is simple and its results agree well with the FEM numerical results. Its accuracy is highly satisfactory for a geotechnical study.

KEY WORDS: tunnel analysis; new implicit method (NIM)

INTRODUCTION

From a general point of view, the problem of the supported tunnel has two different particularities:

- First, it is essentially a three-dimensional problem: close to the tunnel face, the displacement field has a complex shape.
- Secondly, it is a coupled problem concerning the interaction between two different structures for which the geometry and the behaviour are radically distinct: the lining, that can be assimilated to a ring, and the rockmass with its excavation.

The main purpose of this paper is to study accurately this coupled 3D problem under axisymmetrical hypothesis and to quantify the role played by each parameter which describes the system.

Another aim of this research is to propose a new simplified method, in the same way as the well known 'convergence–confinement' method (AFTES[1]) which is able to transform the 3D problem into a plane strain analysis.

CCC 0363–9061/96/090673–18

Received 20 May 1994
Revised 11 December 1995

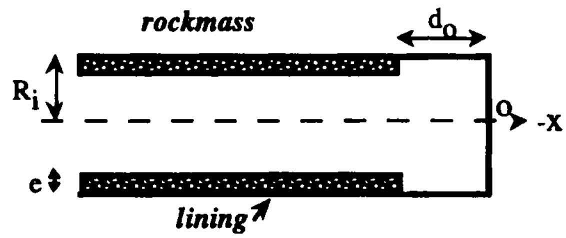

Figure 1. Tunnel modelling

Hereafter, we consider a circular deep tunnel (radius R_i), driven in an homogeneous and isotropic rockmass (Figure 1), initially subjected to a geostatic stress field:

$$\boldsymbol{\sigma}_0 = -P_\infty \mathbf{1} \quad \text{with } P_\infty = \rho g z \quad (z = \text{mean depth}) \tag{1}$$

The lining is a ring of constant thickness e, made of an homogeneous and isotropic material. The lining is set at a distance d_0 from the tunnel face. The face is plane and vertical.

With the above assumptions, the tunnel problem becomes an axisymmetrical problem. Moreover, if the tunnel face is far from the studied section ($x \to \infty$), the displacement field is radial (plane strain condition) $\xi = u(r, t)\mathbf{e}_r$. In this case, all the mechanical parameters describing the structure depend only on the radial distance r to the tunnel axis and the time t.

The interaction between the two previously described structures is now defined by one single scalar parameter: the confinement P_i (radial action of the rockmass on the lining). The associated parameter is the closure of the wall U_i, relative to the variation of the inner radius:

$$U_i = \frac{-u(R_i, t)}{R_i} \tag{2}$$

Thus, we can now define two important curves in the convergence–confinement diagram $P_i - U_i$, that can be considered as a summary of what information we have about both structures:

- The convergence curve C_V gives the internal pressure exerted at the wall of the tunnel P_i versus the convergence of the wall U_i.
- The confinement curve C_F illustrates the relationship between the pressure P_i versus the closure of the lining U_i^S.

If we now draw the C_V and C_F curves on the same diagram P_i–U_i, the equilibrium (P_{eq}, U_{eq}) is given by their intersection, where P_{eq} is the pressure exerted by the rockmass on the lining at the end of the construction process (i.e. $x \to \infty$) and U_{eq} is the total closure of the wall (Figure 2).

This observation allows emphasis on the main coupling parameter of the problem, U_0: the convergence of the tunnel when the lining is set. U_0 represents the initial abscissa of the C_F curve and it depends on the entire tunnel construction process, i.e., for independent time rockmass (elasticity and plasticity), on the rockmass behaviour (C_V), the lining behavior (C_F) and the distance d_0 between the tunnel face and the last ring of lining:

$$U_0 = f(C_V, C_F, d_0) \tag{3}$$

So, in order to achieve the tunnel analysis and use the simplified method, the value of U_0 has to be evaluated.

Numerical methods using three-dimensional (3D) analysis under axisymmetric conditions or not,[2-6] allow calculation with good accuracy of the value of U_0, for a given set of parameters,

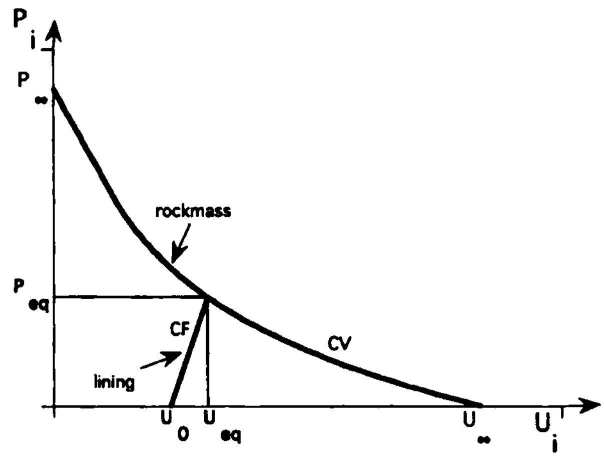

Figure 2. Equilibrium. Definition of U_0

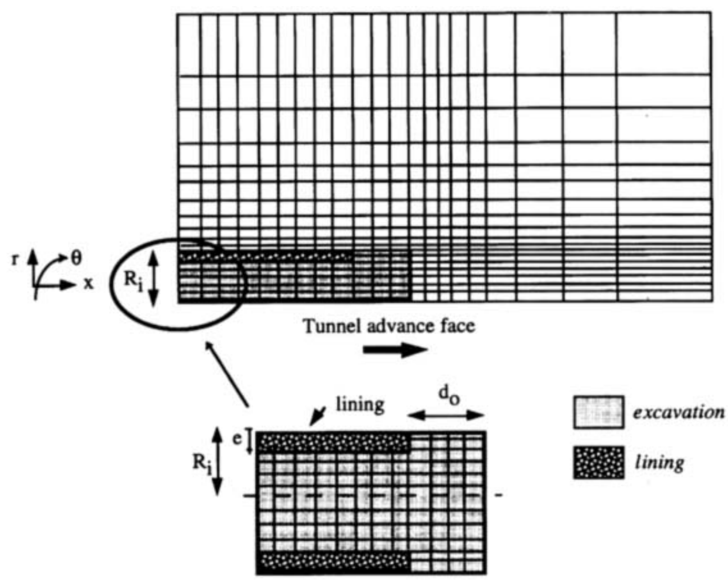

Figure 3. Typical mesh

while simplified methods,[7,8] like 'Convergence–Confinement' method[1,9] or the new implicit one, propose simpler ways to estimate this fundamental parameter U_0.

NUMERICAL APPROACH: 'DEACTIVATION/REACTIVATION METHOD'

The tunnel face advance process is studied here by the 'deactivation/reactivation' method developed in the numerical code 'GEOMEC91' (Bernaud[2]). It is a 3D axisymmetric analysis of tunnelling that takes into account the sequences of excavation and lining placement.

In this model (see Figure 3), the simulation of tunnel construction can be summarized by the following features:

(a) The excavation process is simulated by the 'deactivation' of the stiffness of the elements representing the ground removed during the excavation step (face advance). This is accomplished by a significant reduction of their elastic modulus (E) and Poisson's ratio (ν).
(b) Tunnel support placement using the reactivation process at a specified distance, d_0, from the advancing face. This is calculated by substituting the lining characteristics in the corresponding lining elements. At this moment, the lining elements are stress and strain free.

CONVERGENCE–CONFINEMENT METHOD

The 'Convergence–Confinement' method (C_V–C_F method), proposed by AFTES[1] gives a simple tool to take into account the interaction between both structures and allows to treat the above problem as a plane strain–axisymmetric problem in a plane normal to the axis, where the effects of the surrounding ground are simulated by varying a fictitious pressure P_i^f exerted on the tunnel wall.[9] P_i^f is the internal pressure applied at the wall of the tunnel of infinite length (1D model), to obtain the same value of closure $U_i^f(x)$ corresponding to the three-dimensional calculation of the unlined tunnel (Figure 4). In other words, for a given value of x, the point $P_i^f(x) - U_i^f(x)$ belongs to the convergence curve C_V.

Very often, one can use the parameter λ, varying between 0 and 1:

$$P_i^f(x) = (1 - \lambda(x))P_\infty \tag{4}$$

One must observe that the curves $P_i^f(x)$, $U_i^f(x)$ or $\lambda(x)$ must be calculated for a given behaviour of the rockmass, by using an axisymmetric numerical approach. But many authors,[3,7,10] have proposed explicit formulations for these profiles, in case of elastic or elastoplastic materials.

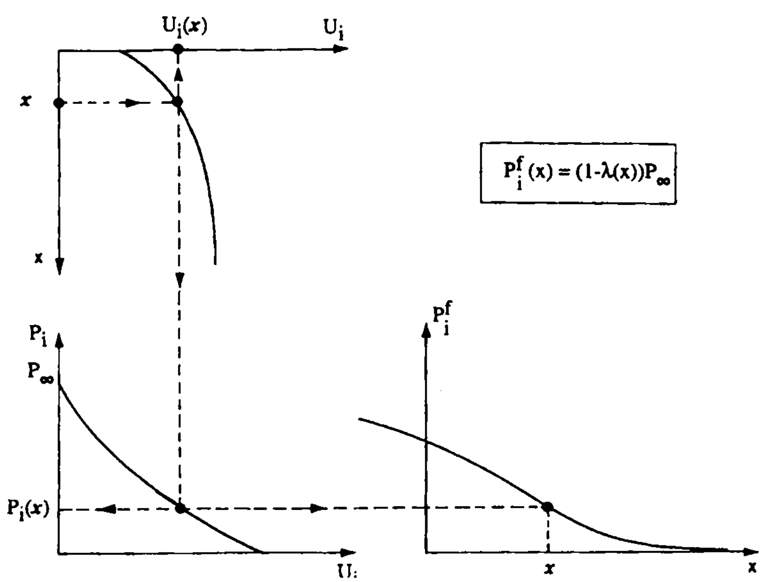

Figure 4. Definition of the fictitious pressure $P_i^f(x)$ and the parameter λ

Introduce a shape function $a(x)$ (varying between 0 and 1):

$$a(x) = \frac{U_i^f(x) - U_i^f(0)}{U_i^f(\infty) - U_i^f(0)} \tag{5}$$

where $U_i^f(0)$ is the convergence of the tunnel face and $U_i^f(\infty)$ is the convergence corresponding to the unlined tunnel with an infinite length. A good approximation of the unlined profile of the convergence is:

$$a^0(x) = 1 - \left(\frac{0{\cdot}84\,R_p}{x + 0{\cdot}84\,R_p}\right)^2 \tag{6}$$

$$U_i^f(0) = 0{\cdot}29\,U_i^f(\infty) \tag{7}$$

Here R_p is the external radius of the plastic zone (in the case of elasticity, $R_p = R_i$).

We have checked that equations (6)–(7) give a good approximation of the convergence profiles of the unsupported tunnel for elastic and perfect elastoplastic behaviours.

The application of the convergence–confinement method is easy (see Figure 6); for a given value of d_0, the curve $P_i^f(x)$ gives the value of $P_i^f(d_0)$ (or $\lambda(d_0) = \lambda_0$ or $U_0 = U_i^f(d_0)$). The knowledge of U_0 allows to place the confinement curve C_F in the P_i–U_i diagram. The intersection between C_F and C_V gives the equilibrium of the tunnel.

The main approximation of this method comes from the hypothesis that the convergence U_0 can be calculated independently of the lining behaviour. This is a major simplification of the interaction problem set at the beginning and can lead sometimes to non-negligible errors as we will see later. In the C_V–C_F method, the convergence U_0 is written as:

$$U_0 = f(C_V, d_0) \tag{8}$$

In order to illustrate the problem, we examine an elastoplastic incompressible rockmass (Tresca criterion and $\nu = 0{\cdot}5$). A non-dimensional analysis shows that the whole problem depends only upon four different parameters:

$$P'_\infty = \frac{P_\infty}{E}, \qquad K'_S = \frac{K_S}{E}, \qquad C' = \frac{C}{E}, \qquad d'_0 = \frac{d_0}{R_i} \tag{9}$$

where E is the Young modulus, C the cohesion and K_S is the stiffness of the elastic lining.

In Figure 5, we have drawn the main results of the numerical analysis (deactivation–reactivation method) for three different cases.

For each case, $P'_\infty = 0.008$ (for example $P_\infty = 4$ MPa, $E = 500$ MPa: quite smooth rockmass 200 m deep).

In Figure 6, the same cases are treated by application of the convergence–confinement method (C_V–C_F).

The main differences between both methods (see Table I) are:

- The deactivation–reactivation method shows that U_0 effectively depends on the lining stiffness: $U_0 = 1{\cdot}73$ per cent for case 2 and $1{\cdot}94$ per cent for case 1; the C_V–C_F method gives the same value for both cases: $U_0 = 2{\cdot}94$ per cent.
- The differences in the value of the lining pressure at the equilibrium, P_{eq}, are significant between both methods. The relative error on P_{eq} can reach 48 per cent (case 3; see Table I).

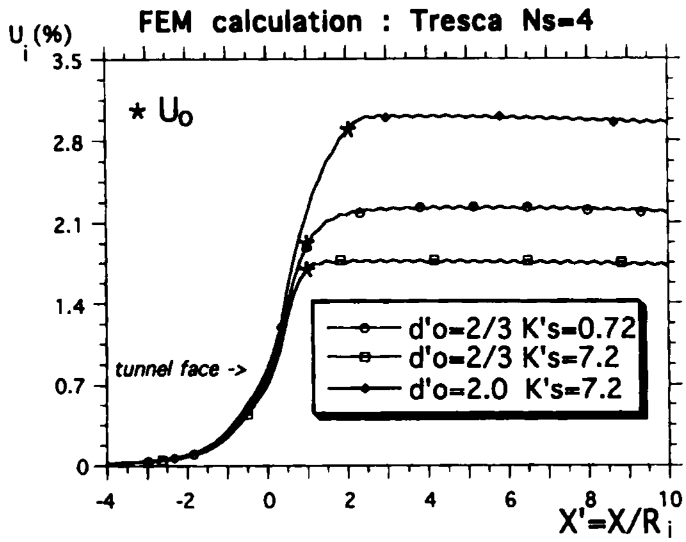

Figure 5. 3D axisymmetric calculation of the lined tunnel

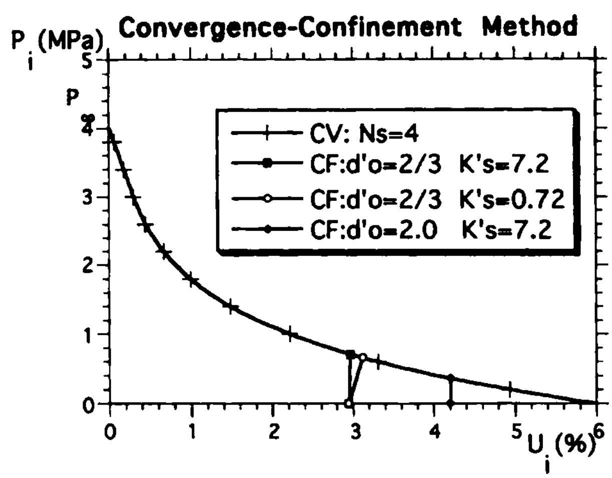

Figure 6. C_F–C_V method used for the calculation of the lined tunnel

- This difference has always the same sign: the C_V–C_F method under estimates the value of the lining pressure at the equilibrium. It is maximal when a very stiff lining is set close to the face.

To sum up, it appears that, in many cases, the application of the convergence–confinement method gives a largely underestimated value of the lining pressure. In fact, the value of the convergence U_0 before the lining is set depends on the rockmass behaviour and on the distance d_0, as predicted by the C_V–C_F method, but also on the stiffness of the lining previously set (Figure 7). So the problem described in the first paragraph, remains a coupled problem.

Table I. Results in plasticity with both methods (numerical calculation and C_V–C_F method)

	Method (1) deactivation/reactivation			Method (2) convergence–confinement			Difference (2 − 1)/1(%)		
Parameters									
Excavation step	$1/3\ R_i$								
d'_0	2/3		2	2/3		2	2/3		2
K'_S	0·72	7·2	7·2	0·72	7·2	7·2	0·72	7·2	7·2
Results									
U_0 (%) ($x = d_0$)	1·94	1·73	2·98	2·94	2·94	4.20	52	70	41
U_{eq} (%)	2·22	1·76	3·00	3·12	2·96	4·21	41	68	40
P'_{eq} ($\times 10^{-3}$)	2·00	2·46	1·39	1·32	1·42	0·72	− 34	− 42	− 48

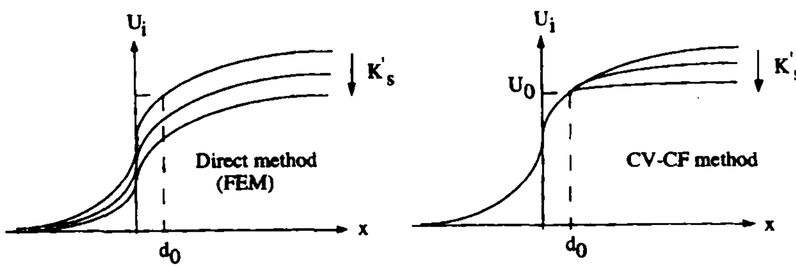

Figure 7. Qualitative influence of the lining stiffness on the convergence profile

'NEW IMPLICIT METHOD' TO STUDY TUNNEL DESIGN

The 'New Implicit Method' (NIM) uses principles similar to those of the Convergence–Confinement method, but it gives a better appreciation of the coupled behaviour between rockmass and lining.

Basic principles of the new method

The aim of the 'New Implicit Method' is to deduce the convergence curve of the supported tunnel $U_i(x)$ from the curve of the unsupported tunnel $U_i^f(x)$, throughout a simple geometrical transformation.

As with the supported tunnel, the 3D FEM calculation that we have performed with the 'deactivation/reactivation method', shows that the relationship

$$a(x) = \frac{U_i(x) - U_i(0)}{U_i(\infty) - U_i(0)}$$

(see Figure 8) presents a shape depending on the lining stiffness. This phenomenon is illustrated in the Figure 9, where several curves $a(x)$ are drawn corresponding to different lining stiffness, for an elastic tunnel.

The classical convergence–confinement method does not take into account the K_S dependence of this parameter $a(x)$. Therefore, we propose another formulation for $a(x)$ that takes into account

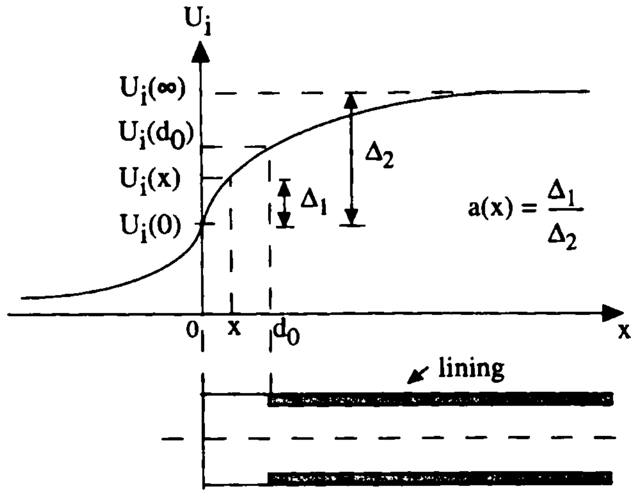

Figure 8. Definition of the function $a(x)$

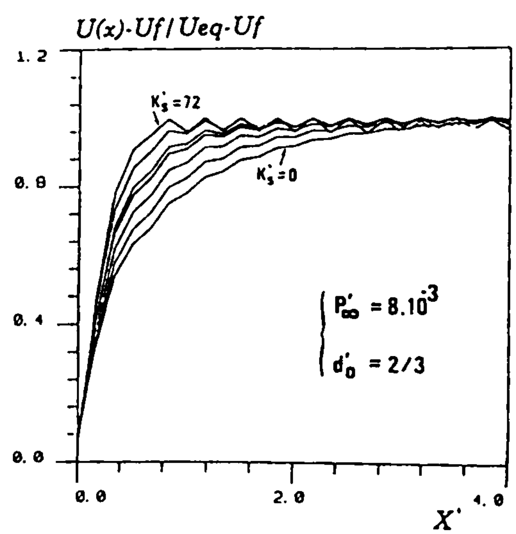

Figure 9. Function $a(x)$ for several K_S'

the phenomenon shown before. We choose to construct $a^S(x)$ (function $a(x)$ for the supported tunnel) by multiplying the x co-ordinate by a scalar α like

$$a^S(x) = a^0(\alpha x) \tag{10}$$

with

$$\alpha = \alpha(K_S') \tag{11}$$

Then, for a given constitutive law of rockmass, the application of the 'NIM' can be summarized by the following steps:

1. Choose the function $a^0(x)$ for the unsupported tunnel. We propose (see equation (6)):

$$a^0(x) = 1 - \left(\frac{0{\cdot}84R'_p}{x' + 0{\cdot}84R'_p}\right)^2 \tag{12}$$

with

$$x' = \frac{x}{R_i}; \qquad R'_p = \frac{R_p}{R_i} \quad (R'_p = 1 \text{ for elastic rockmasses})$$

2. Choose the lining function $\alpha^*(K'_s)$. Then, for the supported tunnel, equation (12) becomes

$$a^S(x') = 1 - \left(\frac{0{\cdot}84}{\alpha^* x' + 0{\cdot}84}\right)^2, \quad \text{with } \alpha^* = \alpha\frac{R_i}{R_p} \tag{13}$$

3. Choose the convergence of the tunnel face $U_f = U_i(0)$.

4. The solution of the problem is then obtained by solving the following equations (see Figure 10):

$$\begin{aligned} P_{eq} &= C_F(U_0, U_{eq}) \\ P_{eq} &= C_V(U_{eq}) \end{aligned} \tag{14}$$

In the system (14), the convergence U_0 is a function of U_{eq} (that explains the name of 'implicit method'). By (5) and (13) we can write:

$$U_0 = a^S(d_0)(U_{eq} - U_f) + U_f \tag{15}$$

Development of the new method

As described before, the NIM requires an evaluation of the lining function α^* and of the convergence U_f at the tunnel face, for each constitutive law of rockmass. We have performed many FEM numerical calculations in order to determine these two functions for incompressible elastic and perfect elastoplastic materials.

Figure 11 gives an example of an elastic numerical calculation with $P'_\infty = P_\infty/E = 0{\cdot}008$; $d'_0 = d_0/R_i = \frac{2}{3}$ and $K'_S = K_S/E = 0{\cdot}072 \rightarrow 72$. In elasticity, the problem of supported tunnel is represented by three independent parameters (P'_∞, K'_S, d'_0) only.

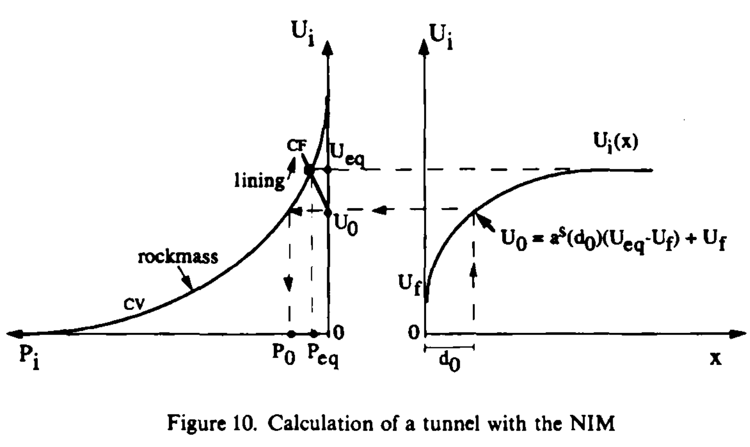

Figure 10. Calculation of a tunnel with the NIM

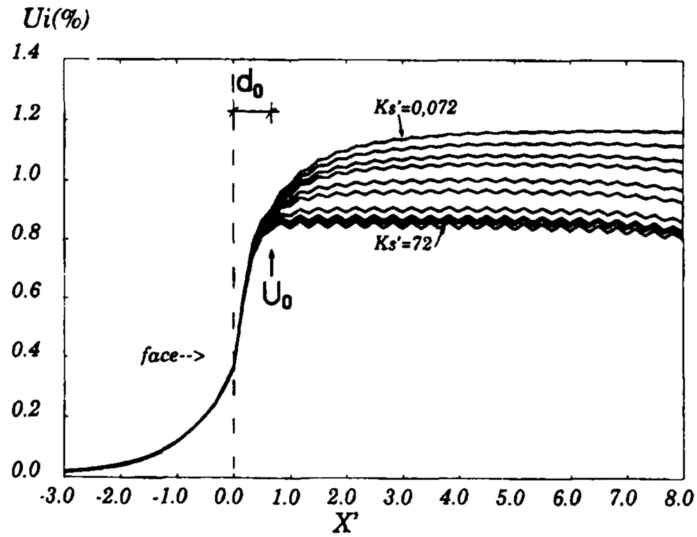

Figure 11. 3D FEM calculations of supported tunnel: elastic rockmass

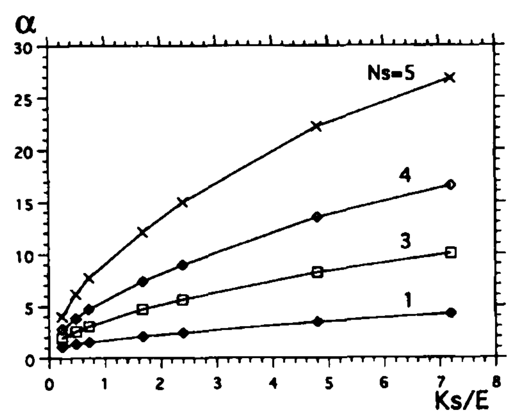

Figure 12. Lining function α for many values of N_S

Figure 12 shows the function α versus K'_S obtained from the FEM numerical calculation in elasticity and plasticity with a Tresca criterion for: $P'_\infty = 0{\cdot}008$; $d'_0 = \frac{2}{3}$; $N_S = P_\infty/C = 1{\cdot}0$; $3{\cdot}0$; $4{\cdot}0$; $5{\cdot}0$ and $K'_S = 0{\cdot}072 \to 7{\cdot}2$. Figure 13 gives the lining function $\alpha^* = (\alpha/R_p)R_i$ corresponding to each α, where it is very interesting to observe that the α^* functions corresponding to different N_S are very similar (N_S = stability coefficient).

As we are interested in constructing a simple method, we propose here to choose a unique α^*_{moy} curve that is able to represent an average behaviour of elastic and perfect elastoplastic (Tresca and Mises) materials.

For this purpose, we give

$$\alpha^*_{\text{moy}} = 1{\cdot}82\sqrt{K'_S} \quad \left(\text{for } K'_S \geqslant \frac{R_i^2}{(1{\cdot}82R_p)^2}\right) \tag{16}$$

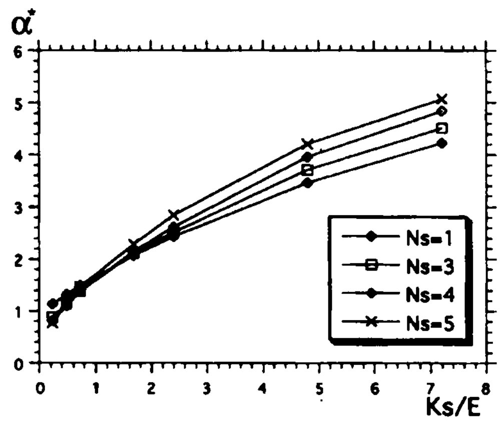

Figure 13. Lining function α^*

for $K_S' < R_i^2/(1{\cdot}82\,R_p)^2$ the unsupported tunnel function is used: $\alpha^*_{moy} = R_i/R_p$.

In the following, we give the lining function α^* for a Coulomb and Hoek–Brown[11,12] rockmasses without dilatancy:

Coulomb ($F = \sigma_1 - \sigma_3 + (K_p - 1)(\sigma_1 - H)$ with $\sigma_1 > \sigma_3 > \sigma_2$ principal stresses)

$$\alpha^* = \alpha^*_{moy} + 0{\cdot}035\phi \tag{17}$$

with ϕ the friction angle in degrees and α^*_{moy} given by (16); $K_p = (1 + \sin\phi)/(1 - \sin\phi)$; $H = C\,\mathrm{cotg}\,\phi$

Hoek–Brown ($F = \sigma_1 - \sigma_3 - \sqrt{(s\sigma_c^2 - m\sigma_1\sigma_c)}$ and $s = 1$)

$$\alpha^* = \alpha^*_{moy} + 0{\cdot}15m \tag{18}$$

Hoek–Brown Modified $\left(F = \sigma_1 - \sigma_3 - \sigma_c\left(-m_b\dfrac{\sigma_1}{\sigma_c}\right)^a\right)$

$$\alpha^* = \alpha^*_{moy} + 0{\cdot}25m_b \tag{19}$$

with m, s, m_b, a, the material constants (see References 11 and 12) and σ_c the uniaxial compressive strength of the rock.

Equations (17)–(19) are useful for $\alpha^* \geqslant R_i/R_p$, otherwise we use $\alpha^* = R_i/R_p$.

It is remarkable that all these functions α^* have been deduced from the lining function α^*_{moy} (valuable for elasticity and Tresca, Mises materials) in a very simple way.

To complete the development of the new method, the convergence at the tunnel face U_f has to be evaluated (see Appendix II).

Finally, with the functions α^* and U_f obtained before, the solution of the coupled problem of supported tunnel is easily obtained by solving the system (14). The expressions of the C_V functions (equation (14)) for the constitutive laws studied here are given in Appendix I.

Application of the new method

So as to check the validity of the new method, we have performed a number of 3D axisymmetric FEM numerical calculations for all the independent parameters of each constitutive law and we have compared the results with the NIM ones.

Figure 14 gives the pressure P'_{eq} versus the lining stiffness K'_S for a Tresca material. For each problem the lining is set up at a distance $d'_0 = \frac{2}{3}$ behind the tunnel face. The resulting average error of this new method is less than 5 per cent for the dimensioning parameters P_{eq} and U_{eq}.

Figure 15 illustrates the difference between the results obtained by the classical convergence–confinement method and the new implicit method for a particular case (Tresca $N_S = 3$; $K'_S = 7{\cdot}2$; $d'_0 = 2/3$; $P'_\infty = 0{\cdot}008$). In this case the error given by the C_V–C_F method on the value of P_{eq} is about -30 per cent and it is clear from Figure 15 that errors are due to the bad evaluation of the convergence at the moment of the lining installation U_0.

Another use of a simplified method is to determine the ratio λ_0 (or the convergence U_0) at the moment of lining installation and then use this value as a good approach to calculate the tunnels

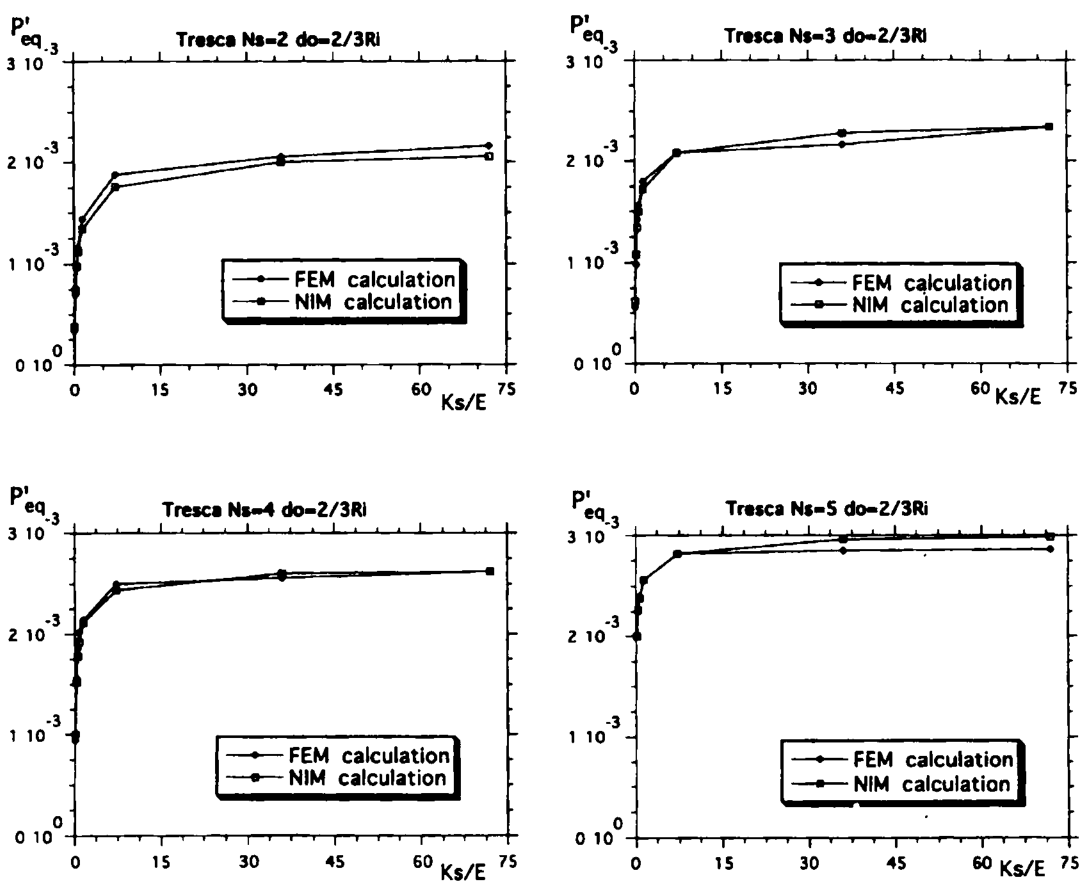

Figure 14. Comparison between FEM numerical calculation and NIM for Tresca rockmass

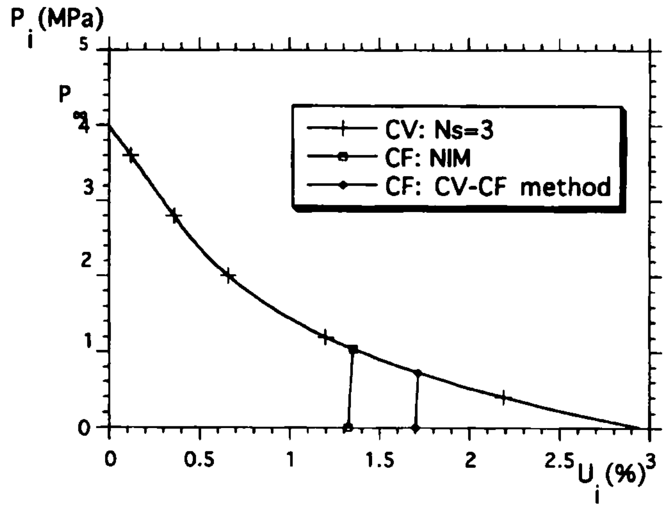

Figure 15. Comparison between C_V–C_F method and NIM

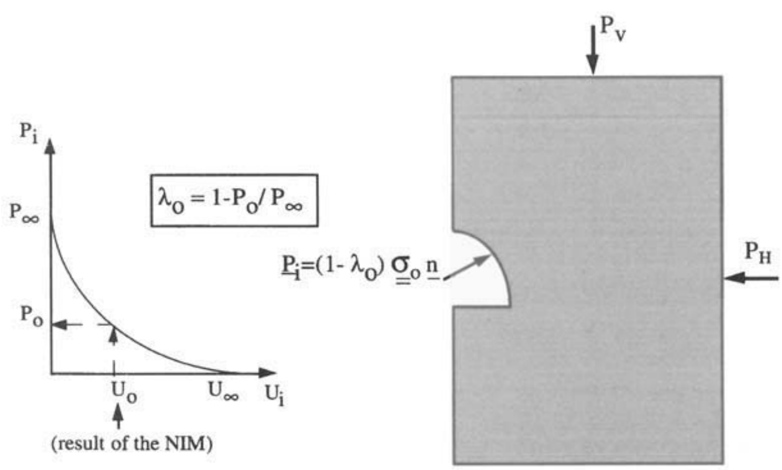

Figure 16. Application of the fictitious pressure (or λ_0) in a plane strain model

in a 2D plane strain condition with a more complex geometry and loading (anisotropic loading for instance). Figure 16 illustrates this approach.

This new method has been checked for a number of independent parameters of supported tunnels (Table II shows some results) for many constitutive laws for the rockmass. The error between the new method and the numerical solution is always less than 10 per cent for the dimensioning parameters P_{eq} and U_{eq}.

Table II. Validity of the new implicit method for many constitutive laws (P'^{c}_{eq}: FEM calculations; P'^{a}_{eq}: NIM calculations)

Constitutive law	P'_{∞}	d'_0	K'_S	P'^{c}_{eq} ($\times 10^{-3}$)	P'^{a}_{eq} ($\times 10^{-3}$)	$\frac{P'^{c}_{eq} - P'^{a}_{eq}}{P'^{a}_{eq}}$ %
Tresca						
$C = 1{\cdot}0$ MPa	0·008	1/3	7·2	2·96	3·0	− 1·8
$C = 1{\cdot}33$ MPa	0·008	1·0	72·0	1·8	1·7	5·5
Coulomb						
$C = 0{\cdot}8$ MPa $\phi = 4°$	0·008	2/3	0·24	1·52	1·50	1·2
$C = 0{\cdot}8$ MPa $\phi = 15°$	0·008	5/3	2·40	0·70	0·62	10·7
$C = 0{\cdot}8$ MPa $\phi = 30°$	0·01	5/3	7·20	0·618	0·62	− 2·0
Hoek–Brown						
$\sigma_C = 1{\cdot}5$ MPa $m = 1$	0·008	2/3	0·72	1·62	1·48	8·4
$\sigma_C = 1{\cdot}5$ MPa $m = 10$	0·008	2/3	0·72	0·52	0·54	− 2·5
$\sigma_C = 5{\cdot}0$ MPa $m = 1{\cdot}0$	0·008	2/3	0·72	0·94	0·90	5·2
$\sigma_C = 5{\cdot}0$ MPa $m = 1{\cdot}0$	0·008	2/3	72·0	2·20	1·94	11·4
Hoek–Brown modified						
$\sigma_C = 2{\cdot}5$ MPa $m_b = 1{\cdot}0$ $a = 0{\cdot}3$	0·008	2/3	0·48	1·30	1·24	5·6
$\sigma_C = 3{\cdot}5$ MPa $m_b = 10$ $a = 0{\cdot}3$	0·008	2/3	0·48	0·24	0·24	0
$\sigma_C = 2{\cdot}5$ MPa $m_b = 2{\cdot}0$ $a = 0{\cdot}5$	0·008	2/3	7·2	1·18	1·22	− 3·5
$\sigma_C = 2{\cdot}5$ MPa $m_b = 1{\cdot}5$ $a = 0{\cdot}4$	0·008	2/3	7·2	1·28	1·38	− 7·2

CONCLUSIONS

It is well-understood that a supported tunnel behaves as a three-dimensional structure for which the strain and the stress fields of the surrounding ground are strongly influenced by details of the technology of excavation and the construction sequences, which can hardly be introduced in numerical models, even when adopting complex 3D FEM.

Nevertheless, for predimensioning purpose it is interesting to use simplified methods for tunnel analysis.

In this paper, we have proposed a new simplified method to study the equilibrium of a lined tunnel driven in a deep rockmass: the New Implict Method (NIM), which can easily be incorporated in a microcomputer, thus constituting a simple tool to perform sensitive studies of tunnel analysis and a useful guide for further complex FEM calculations.

Moreover, if the tunnel verifies the axisymmetric conditions and if the construction sequence presents the ideal configuration studied here (constant tunnel advance rate and the same d_0 for all the sequences), the NIM gives a very accurate response of the problem.

A major improvement of this method, compared to the convergence–confinement method is that it takes into account the dependence of the lining stiffness on the convergence U_0 when the lining is set.

This physical coupling is modelled by introducing a unique explicit lining function α^* which is formulated for many constitutive laws of the rockmass: elasticity and perfect plasticity (Tresca, Mises, Mohr–Coulomb, Hoek–Brown criteria).

The accuracy of the new method is very satisfying for a geotechnical study. The average error is only less than 10 per cent for the major dimensioning parameter: the pressure exerted by the rockmass on the lining when equilibrium is achieved.

APPENDIX I

Solution of the problem at equilibrium

I.a. Elasticity. In this case, the solution of the system (14) is explicit:

$$U_{eq} = \frac{P_\infty + K_s U_f(1 - a^s(d_0))}{K_s(1 - a^s(d_0)) + 2E/3} \tag{1}$$

$$P_{eq} = P_\infty - \tfrac{2}{3} E U_{eq} \tag{2}$$

I.B. Plasticity Tresca. (for $P_{eq} < P_\infty - C$: for $P_{eq} > P_\infty - C$ the rockmass is elastic)
The solution of system (14) is given by

$$\boxed{a \ln U_{eq} + b\, U_{eq} + c = 0}$$

with

$$a = -\frac{C}{K_s}$$

$$b = a^s(d_0) - 1 \tag{4}$$

$$c = (1 - a^s(d_0))U_f + \frac{P_\infty}{K_s} + \frac{C}{K_s}\left\{\ln\left(\frac{1{\cdot}5C}{E}\right) - 1\right\}.$$

For Mises ($F = \sqrt{(3J12)} - 2C$) the solution is given by (14) with C replaced by $2C/\sqrt{3}$.

I.c. Plasticity Coulomb. for $P_{eq} < 2/(K_p + 1)\{P_\infty + H/2(1 - K_p)\}$, otherwise the rockmass is elastic)
The solution of system (14) is

$$\boxed{a\, U_{eq}^{n_1} + b\, U_{eq}^{n_2} + c = 0} \tag{5}$$

with

$$a = \left\{\frac{(K_p + 1)E}{1{\cdot}5(K_p - 1)(P_\infty + H)}\right\}^{(K_p - 1)/2}\left(\frac{K_p + 1}{2}\right)\frac{K_s}{E}(1 - a^s(d_0))$$

$$b = \left\{\frac{(K_p + 1)E}{1{\cdot}5(K_p - 1)(P_\infty + H)}\right\}^{(K_p - 1)/2}\left(\frac{K_p + 1}{2}\right)\left\{U_f \frac{K_s}{E}(a^s(d_0) - 1) + \frac{H}{E}\right\}$$

$$c = -\frac{(P_\infty + H)}{E} \tag{6}$$

$$n_1 = \frac{K_p + 1}{2}$$

$$n_2 = \frac{K_p - 1}{2}$$

I.d. Plasticity Hoek–Brown. (for $P_{eq} < |\sigma_y|$, otherwise the rockmass is elastic)
The solution of system (14) is given by

$$a(\ln U_{eq})^2 + b \ln U_{eq} + c\,U_{eq} + d = 0 \tag{7}$$

with

$$a = \frac{m}{16}$$

$$b = -\frac{m}{8}\left\{\frac{4}{m\sigma_c}\sqrt{\sigma_c^2 - m\sigma_c\sigma_y} + \ln\left(\frac{0{\cdot}75}{E}\sqrt{(\sigma_c^2 - m\sigma_c\sigma_y)}\right)\right\}$$

$$c = K_s\frac{a^s(d_0) - 1}{\sigma_c}$$

$$d = \frac{m}{16}\left\{\frac{4}{m\sigma_c}\sqrt{(\sigma_c^2 - m\sigma_c\sigma y)} + \ln\left(\frac{0{\cdot}75}{E}\sqrt{(\sigma_c^2 - m\sigma_c\sigma_y)}\right)\right\}^2 - \frac{1}{m} + U_f\frac{K_s}{\sigma_c}(1 - a^s(d_0))$$

$$\sigma_y = -\frac{m\sigma_c}{8} - P_\infty + \frac{\sigma_c}{2}\sqrt{\left(\frac{m^2}{16} + \frac{mP_\infty}{\sigma_c} + 1\right)} \tag{8}$$

I.e. Plasticity Hoek–Brown Modified. (for $P_{eq} < |\sigma_y|$, otherwise the rockmass is elastic):

$$\boxed{A(B \ln U_{eq} + C)^{1/(1-a)} + D\,U_{eq} + F = 0} \tag{9}$$

with

$$A = \frac{1}{m_b}$$

$$B = \frac{(a-1)}{2}m_b$$

$$C = \frac{(a-1)}{2}m_b \ln\left\{\frac{E}{0{\cdot}75}\frac{1}{\sigma_c}\left(-m_b\frac{\sigma_y}{\sigma_c}\right)^{-a}\right\} + \left(-m_b\frac{\sigma_y}{\sigma_c}\right)^{1-a}$$

$$D = K_s\frac{a^s(d_0) - 1}{\sigma_c} \tag{10}$$

$$F = U_f K_s\frac{(1 - a^s(d_0))}{\sigma_c}$$

And σ_c given by

$$\frac{\sigma_c}{2}\left(-m_b\frac{\sigma_y}{\sigma_c}\right)^a - \sigma_y - P_\infty = 0 \tag{11}$$

APPENDIX II

Convergence at the tunnel face

In elasticity, the value of U_f is not much affected by the lining stiffness and so, we suggest the following equation (valable in elasticity only):

$$U_f = 0{\cdot}27 U_\infty \quad \text{with} \quad U_\infty = \frac{1+\nu}{E} P_\infty \tag{12}$$

U_∞ is the convergence of the unsupported tunnel far from the tunnel face.

In plasticity, a more significant variation of U_f with respect the lining stiffness K'_s is observed. Nevertheless, with the following, we propose an average value of the convergence U_f (between a high and a low value of K'_s and for many distances d'_0) for each constitutive law.

For Tresca, Mises, Coulomb and Hoek–Brown, we suggest:

$$\frac{U_f}{U_\infty} = 0{\cdot}413 - 0{\cdot}0627 N_s, \quad 1 < N_s \leqslant 5 \tag{13}$$

with

$$N_s = \frac{2P_\infty}{R_c} \quad \text{and} \quad R_c = \frac{2C\cos\phi}{1-\sin\phi} \quad \text{for Tresca, Mises } (\phi = 0) \text{ and Coulomb criteria}$$

$$N_s = \frac{2P_\infty}{\sqrt{(\sigma_c^2 - m\sigma_c\sigma_y)}}; \qquad \sigma_y = \frac{-m\sigma_c}{8} - P_\infty + \frac{\sigma_c}{2}\sqrt{\left(\frac{m^2}{16} + \frac{mP_\infty}{\sigma_c} + 1\right)} \quad \text{for}$$

Hoek–Brown criterion (14)

For Hoek–Brown Modified, equation (13) is no longer worthy, so we suggest:

$$\frac{U_f}{U_\infty} = 0{\cdot}75 - 0{\cdot}5a - 0{\cdot}133 N_s \tag{15}$$

$$N_s = \frac{2P_\infty}{\sigma_c(-\mathrm{m_b}\sigma_y/\sigma_c)^a} \tag{16}$$

where σ_y is obtained by solving the following equation:

$$\frac{\sigma_c}{2}\left(-m_b\frac{\sigma_y}{\sigma_c}\right)^a - \sigma_y - P_\infty = 0 \tag{17}$$

For the above constitutive laws (with $\nu = 0{\cdot}5$) the exact value of U_∞ is reminded below:

Tresca

$$U_\infty = \frac{1{\cdot}5}{E} C \exp(N_s - 1) \tag{18}$$

Coulomb

$$U_\infty = \frac{1{\cdot}5}{E}\frac{K_p - 1}{K_p + 1}(P_\infty + H)\left\{\frac{2}{K_p+1}\frac{P_\infty + H}{H}\right\}^{2/(K_p-1)} \quad \text{with } H = C\operatorname{cotg}\phi \tag{19}$$

Hoek–Brown

$$U_\infty = \frac{0{\cdot}75}{E}\sqrt{(\sigma_c^2 - m\sigma_c\sigma_y)}\exp\left\{\frac{-4(\sigma_c - \sqrt{(\sigma_c^2 - m\sigma_c\sigma_y)})}{m\sigma_c}\right\} \tag{20}$$

Hoek–Brown modified

$$U_\infty = \frac{0{\cdot}75}{E}\sigma_c\left(-m_b\frac{\sigma_y}{\sigma_c}\right)^a \exp\left\{\frac{2}{(1-a)m_b}\left(-m_b\frac{\sigma_y}{\sigma_c}\right)^{1-a}\right\} \tag{21}$$

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